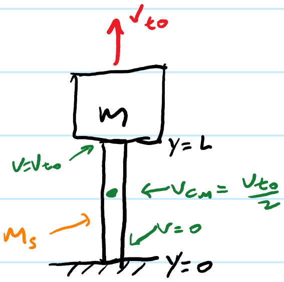
CM → center of mass



at take-off

$$K_{tot} = \underbrace{\frac{1}{2} m v_{to}^2}_{\text{K.E. of load mass}} + \underbrace{\frac{1}{2} m_s \left(\frac{v_{to}}{2}\right)^2}_{\text{K.E. of spring CM translation}} + \underbrace{\int_0^L \frac{1}{2}\frac{m_s}{L}\left(v(y) - v_{cm}\right)^2 dy}_{\text{K.E. of spring about its CM (vibrational K.E.)}}$$

If $v(y) = v_{to}\frac{y}{L}$, $\left(v_{cm} = v_{to}/2\right)$

$$\text{K.E. of spring about CM} = \int_0^L \frac{1}{2}\frac{m_s}{L}\left(v_{to}\frac{y}{L} - \frac{v_{to}}{2}\right)^2 dx$$

$$= \frac{1}{6}\frac{m_s L}{L v_{to}}\left(v_{to}\frac{y}{L} - \frac{v_{to}}{2}\right)^3\Bigg|_{y=0}^{y=L}$$

$$= \frac{1}{6} m_s v_{to}^2 \left(\left(1 - \frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^3\right)$$

$$\text{K.E of spring about CM} = \frac{1}{24} m_s v_{to}^2 = \frac{1}{12}\left(\frac{1}{2} m_s v_{to}^2\right)$$

$$\therefore\ K_{tot} = \underbrace{\frac{1}{2} m v_{to}^2}_{\text{K.E. load mass}} + \underbrace{\frac{1}{4}\left(\frac{1}{2} m_s v_{to}^2\right)}_{\text{K.E translational of spring CM}} + \underbrace{\frac{1}{12}\left(\frac{1}{2} m_s v_{to}^2\right)}_{\text{K.E. vibrational about spring CM}}$$

$$K_{tot} = \frac{1}{2} m v_{to}^2 + \frac{1}{2}\left(\frac{m_s}{3}\right) v_{to}^2 \qquad \left(\frac{1}{4} + \frac{1}{12} = \frac{1}{3}\right)$$

$$K_{tot} = \frac{1}{2}\left(m + \frac{m_s}{3}\right) v_{to}^2$$